**GENERALIZED OF PRIMARY AVOIDANCE THEOREM FOR COMMUTATIVE RINGS**

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**الخلاصة**

في هذا البحث قمنا بدراسة تغطية المثاليات بمجاميع مشاركة لمثاليات ابتدائية وحصلنا على اعمام لمبرهنة تجنب الابتدائيات في الحلقات والتي برهنت في .

**Abstract**

In this paper we study covering ideals by Cosets of primary ideals and we get a generalized the primary avoidance theorem in the rings which it has been proved in .

**1.Introduction**

denotes a commutative ring are Throughout this paper,

ideals of and are elements of we call a covering

efficient if no Coset is superfluous .A special case is when all are equal ,say for We can get and this is a coset efficiently covered by a union of ideals.

In this paper we will replace the Cosets of prime ideals by Cosets of primary ideals and we gave a generalize to the primary avoidance theorem in rings,

**2. Basic properties**

We introduce the following concepts.

**Definition 2.1:**

Let are ideals of ,and are elements of , we call a covering efficient if no coset is superfluous. Analogously we shall say that, is an efficient union if none of the coset may be excluded.

**Remarks 2.2:**

1. Every covering (union) of Cosets can be efficient by deleting unnecessary terms.

2. In the case for all the relation

equivalent,

and we get a Coset covering by the ideals .

3. Every ideal in can be considered as a Coset since .

The following example explain that a covering by two Cosets may be efficient.

**Example 2.3**

In the ring ,the ideal is efficient union of the Cosets and .

**Definition 2.4: [ 2 ]**

An ideal of a ring is said to be a primary ideal if with implies for some positive integer .

**Propositions 2.5: [ 2 ]**

1. Every prime ideal is a primary ideal.
2. If is a primary ideal, then the ideal is prime .
3. for every ideal
4. If P is prime ideal , then or .

**Lemma 2.6**

Let be an efficient covering of by Cosets, where .Then but for all

**Proof.** See [ 4 ].

As an application for lemma 2.6, we get the following proposition.

**Proposition 2.7**

Let ,be an efficient covering with . Then no is prime.

**Proof.** See [ 4 ]

The following theorem is a generalize to the prime avoidance theorem in rings.

**Theorem 2.8**

Let , be a covering such that at least of the ideals are prime, then for some .

**Proof.** See [ 4 ]

The following proposition is due to Gilmer.(Lemma 2).

**Proposition 2.9.**

Let be a covering where ,,…, are pairwise distinct prime ideals then , for some k .

**Poof.** See

**Theorem 2.10 :**

Let be an efficient covering then,

for some .

**Proof .** See [ 4 ]**.**

**Theorem 2.11**

Let be an efficient covering where ,

Then no is primary .

**Proof .** See [ 1 ]**.**

Now, we give the primary avoidance theorem in the rings, [ 1 ]

**Theorem 2.12**

Let be a covering of ideals and suppose that at least ,of the ideals are primary ,then for some .

**Proof .** See [ 1 ].

**Theorem 2.13**

Let and be an ideals in such that :

where are primary ideals, then either or for some .

**Poof.** See

**3. Main Results .**

In this section we given the general case of the primary avoidance

Theorem and some applications.

The following theorem is a generalize to the proposition(2.7).

**Theorem 3.1**

Let  be an efficient covering with . Then no is primary ideal.

**Proof .** Assume that is primary ideal for some .

By remark (2.2) the covering,

is efficient, by lemma(2.6), we get and for all Since is primary ideal then is prime ideal.(prop.2.5).

Thus and . Thus and for some . Now, , thus there exists i.e. and . But , which a contradiction.

Thus none is primary ideal.

The following theorem is a generalize to the primary avoidance in the rings (Theorem 2.12).

**Theorem 3.2**

Let be a covering such that at least of the ideals are primary then , for some .

**Proof.** We may assume that the covering is efficient.

If , then this efficient cover contain primary ideal which a contradiction with theorem(3.1).Thus , i.e. , then ,

Then , then . It is clear that .

Thus .

The following theorem is a generalize to the theorem(3.1).

**Theorem 3.3**

Let be an efficient covering, where ,then none is primary ideal.

**Proof.** Assume that is primary ideal for some .

By lemma(2.6), we get and .Since is primary then by (prop.2.5), is prime ideal, then and .Then and for some . Since , then ,then which a contradiction. Thus non is primary ideal.

The following theorem is a generalize to the theorem (3.2).

**Theorem 3.4**

Let a covering such that at least of the ideals are primary, then there exists , such that .

**Proof.** We may assume that the covering is efficient. If ,then this efficient cover contain a primary ideal which is a contradiction with theorem(3.3).Thus i.e., then , then , then , then , thus .

The following theorem is a generalize to the proposition(2.9). **Theorem 3.5**

Let be a covering where are pair wise distinct primary ideals then , for some .

**Proof.** If ,it follows from theorem(3.4) that the covering cannot be efficient. Thus after reducing to an efficient covering only one term remain, say . Hence .

The following theorem is application for theorem (3.4) and more general to the theorem (2.13).

**Theorem 3.6**

Let and be an ideals in such that :

, where are primary ideals, then either or for some .

**Proof .** Put .

Because every ideal can be considered as a Coset.

It clear that .

If , then by theorem(3.4) for some .

**References**

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