



Edge Domination in Web Graph

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Abstract: Let $\gamma^e(G)$ be the edge domination number of a graph. A “web graph” $W(s, t)$ is obtained from the Cartesian product of cycle graph of order s and path graph of order t . In this paper, edge domination number of the web graph is determined.

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1 Introduction

Let $G = (V, E)$ be a graph with its vertex set $V = V(G)$ and edge set $E = E(G)$. In a graph G an edge $e \in E$ is said to be incident with vertex $v \in V$ if the vertex v is an end vertex of the edge e . Two edges of a graph G are adjacent if they are distinct and have a common vertex. The open neighborhood of an edge $e \in E(G)$ denotes as $N_G(e)$ is the set of all edges that adjacent to e while the closed neighborhood of e is the set $N_G[e] = N_G(e) \cup \{e\}$. We use $|D|$ to denote the cardinality of a set D .

Mitchell and Hedetniemi [6] introduced the definition of “edge domination in the graphs”.

A subset D of the edge set E in graph G is called an *edge dominating set* of G if every edge not in D is adjacent to some edge in D . The “*edge domination*

number” of a graph G which we denote by $\gamma^e(G)$ is the minimum cardinality taken over all edge dominating sets of G . In this work, edge domination of the “web graph” is calculated. Haynes et.al [4] gave the treatment of the “Fundamentals of Domination” in graph theory. For more details on several advanced topics about domination in graphs see [1-2, 5, 7-11]. Any term or notation about the graph in this paper can be found in [3].

Remark 1.1. (i) $\gamma^e(C_n) = \left\lceil \frac{n}{3} \right\rceil$, for $n \geq 3$.

(ii) $\gamma^e(P_n) = \left\lceil \frac{n-1}{3} \right\rceil$, for $n \geq 2$

Definition 1.2. Cartesian product: The “Cartesian product” $G_1 \times G_2$ of the two graphs G_1 and G_2 is the graph G having vertex set $V(G_1) \times V(G_2)$ and two vertices (u_1, u_2) and (v_1, v_2) of G are adjacent if and only if either $u_1 = v_1$ and $u_2v_2 \in E(G_2)$ or $u_2 = v_2$ and $u_1v_1 \in E(G_1)$.

Definition 1.3. A “web graph” $W(s, t)$ is obtained from the “Cartesian product” of cycle graph of order s and path graph of order t .

2 Main results

Web graph

In this work we are describing the “web graph” in terms of cycles and paths. In this graph there are r copies of cycles C_s as shown by dashed lines and s copies of paths P_r as shown by bold lines in Figure 1 as an example. In general, we describe these cycles and paths as follows.

$P_t^i \equiv v_i^1 e_1 v_i^2 e_2 \dots v_i^{s-1} e_{s-1} v_i^s$, where $e_j = v_i^j v_i^{j+1}$, $j = 1, 2, \dots, t - 1 \forall i = 1, 2, \dots, s$, and

$C_s^j \equiv v_1^j e_1 v_2^j e_2 \dots v_{s-1}^j e_{s-1} v_s^j e_s v_1^j$, where $e_i = v_i^j v_{i+1}^j$

$i = 1, 2, \dots, s - 1$ and $e_s = v_s^j v_1^j \forall j = 1, 2, \dots, t$.

We labeled the vertices of this graph by v_i^j where j represents the cycle label starting from the inside out, and i is the vertex label of s of j^{th} cycle ordering clockwise. While i represents the path label ordering clockwise, and j is the label of vertex in i^{th} path starting from the inside out. For example, see Figure1.

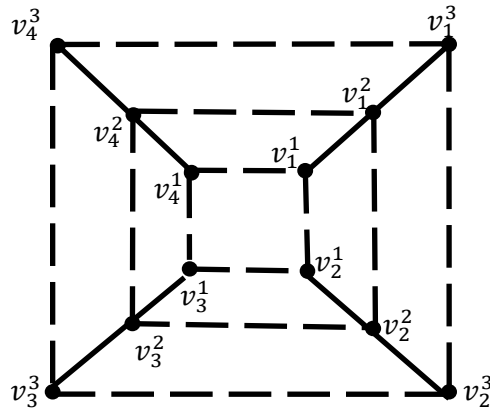


Figure 1: $W(4, 3)$

Theorem 2.1. For G is a web graph $W(s, t)$, where t is even, the edge domination number is

$$\gamma^e(G) = \left\lfloor \frac{s}{2} \right\rfloor \left\lfloor \frac{t-1}{2} \right\rfloor + \left\lfloor \frac{s}{2} \right\rfloor \left\lfloor \frac{t-1}{3} \right\rfloor.$$

Proof. To get the minimum number of edges so that we can dominate all edges of the graph, we must search the edges which have maximum closed neighborhood. There are three types of edges that depend on the cardinal number of closed neighborhood as follows.

$$|N[e]| = \begin{cases} 5, & \text{if the edge } (e) \text{ belongs to the } C_s^1 \text{ or } C_s^t \text{ of the graph} \\ 6, & \text{if the edge } (e) \text{ belongs to the terminal edges of any path} \\ 7, & \text{otherwise} \end{cases}$$

Thus, we get the minimum dominating set when we choose the edges from the paths by the following method.

We choose and let edges, from P_t^i in alternating way, for i is odd. By this choice, all edges of these paths as well as all edges of the cycles are dominated (as an example, see bold edges in Figure 2).

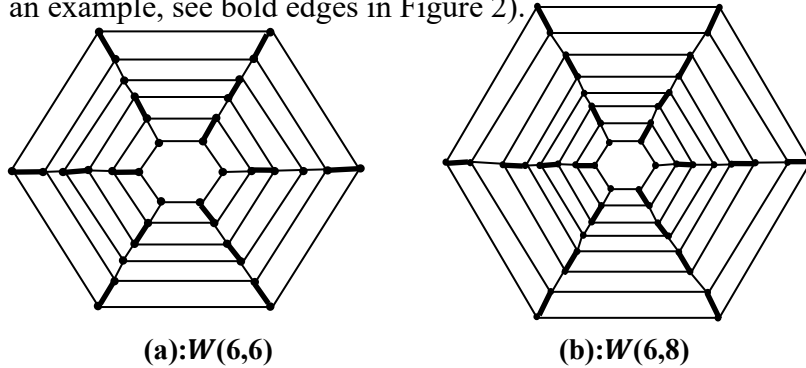


Figure 2

Let D_1 be the set for the chosen dominating edges. Therefore, we can represent this set by

$$D_1 = \bigcup_{i=1}^{\lfloor \frac{s}{2} \rfloor} \left\{ \bigcup_{j=1}^{\lfloor \frac{t}{2} \rfloor} v_{2i-1}^{2j-1} v_{2i-1}^{2j} \right\}, \text{ It is clear that, } |D_1| = \left\lfloor \frac{s}{2} \right\rfloor \left\lfloor \frac{t-1}{2} \right\rfloor.$$

Now, the edges which are not dominated by the set D_1 are the edges of P_t^i , for i is even. So, let $D_2 = \bigcup_{i=1}^{\lfloor \frac{s}{2} \rfloor} \left\{ \bigcup_{j=0}^{\lfloor \frac{t-1}{2} \rfloor} v_{2i}^{2+3j} v_{2i}^{3+3j} \right\}$, be the set that dominates the edges of P_t^i , for i is even. By Remark 1.1(ii), we have $\gamma^e(P_t^i) = \left\lfloor \frac{t-1}{3} \right\rfloor$ for i is even (as an example, see bold edges in Figure 2).

So, it is clear that, $|D_2| = \left\lfloor \frac{s}{2} \right\rfloor \left\lfloor \frac{t-1}{3} \right\rfloor$. Therefore, the set $D = D_1 \cup D_2$ is the dominating set of the “web graph” with minimum cardinality. Thus, we get the result.

Theorem 2.2. For G is a web graph $W(s, t)$, where t is odd, the edge domination number is

$$\gamma^e(G) = \left. \begin{cases} \frac{s}{2} \left(\left\lceil \frac{t-1}{2} \right\rceil + \left\lceil \frac{t-1}{3} \right\rceil \right), & \text{if } s \text{ is even and } t \not\equiv 1 \pmod{3} \\ \left\lceil \frac{s}{2} \right\rceil \left\lceil \frac{t-1}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil \left\lceil \frac{t-1}{3} \right\rceil + \left\lceil \frac{s}{3} \right\rceil, & \text{if } t \equiv 1 \pmod{3} \\ \left\lceil \frac{s}{2} \right\rceil \left\lceil \frac{t-1}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil \left\lceil \frac{t-1}{3} \right\rceil + 1, & \text{if } s \text{ is odd and } t \not\equiv 1 \pmod{3} \end{cases} \right\}$$

Proof: In the same manner in Theorem 2.1, the set $D_1 \cup D_2$ dominates all edges of G when s is even and $t \not\equiv 1 \pmod{3}$ (as an example, see bold edges in Figure 3). Otherwise there are two cases as follows.

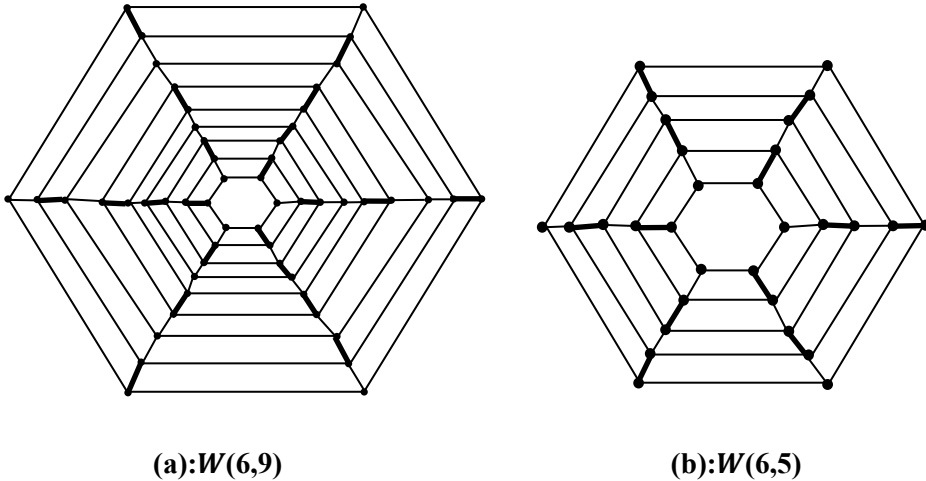


Figure 3

Case 1. If $t \equiv 1 \pmod{3}$, the set D_1 dominates all edges of P_t^i , for i is odd and all edges in every cycle C_s^j , $j = 1, \dots, t-1$ and D_2 dominate the edges of P_t^i , for i is even.

The edges of C_s^t are not adjacent to any edges of D_1 and D_2 , so we can dominate the last C_s^t cycle by $\left\lceil \frac{s}{3} \right\rceil$ edges by using Remark 1.1(i), (as an example, see bold edges in Figure 4). Thus, we get the result.

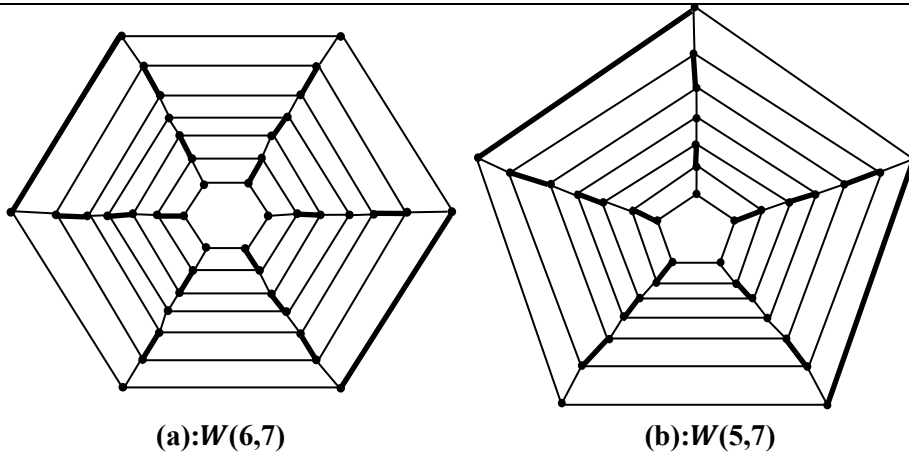


Figure 4

Case 3. If s is odd and $t \not\equiv 1 \pmod{3}$, then in the same manner in the previous case the set D_1 dominates all edges of P_t^i , for i is odd and all edges in every cycles C_s^j , $j = 1, \dots, t$ except one edge (e) in last cycle C_s^t .

D_2 dominates the edges of P_t^i , for i is even.

So, the set $D_1 \cup D_2 \cup \{e\}$ dominates all edges of G (with minimum cardinality) in this case (as an example, see bold edges in Figure 5).

Therefore we get the result.

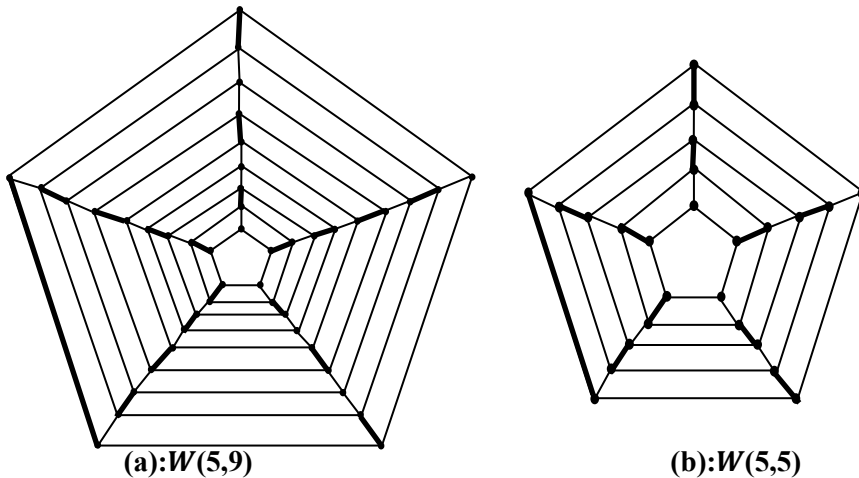


Figure 5



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