Characteristics of Soft Tychonoff Spaces with New Soft Separation Axioms

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Abstract

The aim of this work is to investigate and study soft Tychonoff space and some new soft spaces such as soft $PR-T_3$, soft $SCR-T_3$, soft $PN-T_4$, soft $SCN-T_5$, soft S^2CR-T_3, soft $SPR-T_3$, soft $SPN-T_4$ and soft S^2CN-T_5 in this work the relationships between these new soft spaces such as soft public regular, soft public normal, soft strongly completely regular, soft semi completely normal and with well known axioms soft T_3, soft T_4 and soft T_5 are obtained, Further, the relationships between these new soft spaces with each other are studied. We prove that each soft $SCR-T_3$ space is soft Tychonoff space, also in this paper we show that each soft T_3 space is soft $PR-T_3$, each soft T_4 space is soft $PN-T_4$ and each soft T_5 space is soft $SCN-T_5$. Finally, depending on the above soft spaces and properties of the soft fuzzy spaces, the new types of soft fuzzy spaces can be introduced like Soft fuzzy public regular, Soft fuzzy strongly completely regular, and others.

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1. Introduction

The notation of soft sets is a novel concept was first introduced by Molodtsov [18] is a completely new approach for modeling vagueness and uncertainty. In [24], Molodtsov applied successfully in directions like, operations research, Riemann-integration, Perron integration, smoothness of functions, probability and theory of measurement. Some important applications of soft sets are in information systems and decision making problems can be seen ([15],[16],[17]). In recent years, after presentation of the operations of soft sets theory many interesting applications of soft set theory have been studied increasingly [10, 11, 12, 13, 14]. In [19] Min investigated some properties of such soft separation axioms. After that Kandil et al. [20] introduced new soft separation axioms based on the soft semi open sets which are more general than of the open soft sets.

The aim of this paper is to investigate and study soft Tychonoff space and some new soft spaces such as soft $PR-T_3$, soft $SCR-T_3$, soft $PN-T_4$, soft $SCN-T_5$, soft S^2CR-T_3, soft $SPR-T_3$, soft $SPN-T_4$ and soft S^2CN-T_5. Further, in the present work, the relationships between these new soft spaces such as soft public regular, soft public normal, soft strongly completely regular, soft semi completely normal and with well known axioms soft T_3, soft T_4 and soft T_5 are obtained, Moreover, the relationships between these new soft spaces with each other are studied. We prove that each soft $SCR-T_3$ space is soft Tychonoff space, also in this paper we show that each soft T_3 space is soft $PR-T_3$, each soft T_4 space is soft $PN-T_4$ and each soft T_5 space is soft $SCN-T_5$. Further, the interesting ideas of soft set theory have been expanded and studied by embedding the notations of fuzzy sets. In another side, in this paper depending on the above spaces and properties of soft fuzzy spaces, the new types of soft fuzzy spaces can be introduced like soft fuzzy public regular, soft fuzzy strongly completely regular, etc,. Finally, some suggestions for future work are laid out.

2. DEFINITIONS AND NOTATIONS

The following definitions have been used to obtain the results and properties developed in this paper.

Definition 2.1 ([4], [5], [12]). A pair (F, A) is called a soft set (over U) where F is a mapping $F: A \to P(U)$. That means the soft set is a parameterized collection of subsets of the set U. In another side, any set $F(e)$, $e \in E$, may be considered from this collection as the set of $e-$ approximate elements of the soft set, or as the set of e-elements of the soft set (F, A). Therefore, we can consider that this soft set can be not a set. Let (F, A) and (G, B) be two soft sets over the common universe U, then (F, A) is called a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \tilde{\subseteq} (G, B)$. Further, we say that (F, A) is a soft superset of (G, B), if (G, B) is a soft subset of (F, A) and we say that they are soft equal if each one is a soft subset of other. Moreover, (F, A) is called a null soft set, denoted by $\Phi = (\phi, \phi)$, if for each $F(e) = \phi, \forall e \in A$. Also, it is denoted by (U, E) and called universal soft set, if for each $F(e) = U, \forall e \in A$. The family of all soft sets (F, A) over a universe U and the parameter set A is denoted by $SS(U_A)$.

Definition 2.2: ([9], [13]) Let (F, A) and (G, B) be two soft sets over X, then their union is the soft set (H, C), where $C = A \bigcup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A - B$, $G(e)$ if $e \in B - A$, $F(e) \bigcup G(e)$ if $e \in A \bigcap B$. We write $(F, A) \coprod (G, B) = (H, C)$. Further, [8] for any two soft sets (F, A) and (G, B) over X their intersection is the soft set (H, C) over X, and we write $(H, C) = (F, A) \prod (G, B)$, where $C = A \bigcap B$, and $H(e) = F(e) \bigcap G(e)$ for all $e \in C$.

$(F, A) \in SS(U_A)$ (F, A) is a soft point in (U, A) $x \in U$ and $e \in A$ satisfy $F(e) = \{x\}$ $F(e') = \phi$, $\forall$ $e' \in A - \{e\}$ (F, A) by e_x. The soft point e_x is said to be belonging to the soft set (G,A) and is denoted by $e_x \tilde{\in} (G,A)$, if for the element $e \in A$ and $F(e) \subseteq G(e)$.

Definition 2.4: ([7]) Let (F,A) be a soft set over U. The complement of (F,A) with respect to the universal soft set (U,E), it is written as $(F,A)^c$ and defined by (F^c,D), where
, and for all $e \in D$, $D = E \setminus \{e \in A \mid F(e) = U\} = \{e \in A \mid F(e) = U\}^c$

$$F^c(e) = \begin{cases} U \setminus F(e)\ ,\ if\ \ e \in A \\ U\ ,\ \ Otherwise\ . \end{cases}$$

Definition 2.5: ([3], [2]). Assume that τ is a collection of soft sets over U. Then τ is said to be a soft topology on U if τ such that the following axioms:

(i) Φ, (U,E) belong to τ.

(ii) For any collection $\{(F_i,A_i)\}_{i\in I}$ of soft sets in τ, such that $\coprod_{i\in I}(F_i,A_i)$ belongs to τ.

(iii) For any pair of soft sets in τ, their intersection belongs to τ.

The triplet (U,E,τ) is said to be a soft topological space over U. Further, any member in τ is called soft open set in U and its complement is called soft closed set in U.

Definition 2.6 Let (U,E,τ) be a soft topological space and $(F,A) \in SS(U_E)$. Then

(1) The soft closure of (F,A), denoted by $cl(F,A)$ is the intersection of all closed soft super sets of (F,A) (i.e,
$cl(F,A) = \prod\{(G,B) : (G,B)\, is\, soft\ \ closed\ \ set\ \ and\ \ (F,A) \tilde{\subseteq} (G,B)\}$) ([22]).

(2) The soft interior of (F,A), denoted by $\text{int}(F,A)$ is the union of all open soft subsets of (F,A) (i.e,
$\text{int}(F,A) = \coprod\{(G,B) : (G,B)\, is\, soft\ \ open\ \ set\ \ and\, (G,B) \tilde{\subseteq} (F,A)\}$) ([22]).

(3) (F,A) is called soft semi open set iff $(F,A) \tilde{\subseteq} cl(\text{int}((F,A))$ ([21]).

Definition 2.7 Assume that (U,E,τ) is a soft topological space over U and $x,y \in U$. Then (U,E,τ) is called a soft T_1-space if there exist two soft open sets (F,A) and (G,B) such that $e_x \tilde{\in} (F,A)$ and $e_y \tilde{\notin} (F,A)$ and $e_y \tilde{\in} (G,B)$ and $e_x \tilde{\notin} (G,B)$ ([22]). Moreover, (U,E,τ) is called a soft semi T_1-space if there exist two soft semi open sets (F,A) and (G,B) satisfy $e_x \tilde{\in} (F,A)$ and $e_y \tilde{\notin} (F,A)$ and $e_y \tilde{\in} (G,B)$ and $e_x \tilde{\notin} (G,B)$ ([21]).

Definition 2.8 ([22]). Assume that (U,E,τ) is a soft topological space over U , and let (F,A) be a soft closed set in U and $x \in U$ such that $e_x \tilde{\notin} (F,A)$. If there exist two soft open sets (G,B) and (H,C) such that $e_x \tilde{\in} (G,B)$, $(F,A) \tilde{\subseteq} (H,C)$ and $(G,B) \prod (H,C) = \Phi$, then (U,E,τ) is said to be a soft regular space. A soft regular T_1-space is called a soft T_3-space.

Definition 2.9 ([22]). Assume that (U,E,τ) is a soft topological space over U and let $(F,A),(G,B)$ be two soft closed sets such that $(F,A) \prod (G,B) = \Phi$. If there exist two soft open sets (H,C) and (K,D) such that $(F,A) \tilde{\subseteq} (H,C),(G,B) \tilde{\subseteq} (K,D)$ and $(H,C) \prod (K,D) = \Phi$, then (U,E,τ) is called a soft normal space. A soft normal T_1-space is called a soft T_4- space.

Definition 2.10 ([28]). Let (U,E,τ) be a soft topological space over U and let $(F,A),(G,B)$ be two non-empty soft subsets of (U,E). Then we say that $(F,A),(G,B)$ are two separated sets if $(F,A) \prod cl((G,B)) = \Phi$ and $cl((F,A)) \prod (G,B) = \Phi$.

Definition 2.11 ([28]). A soft topological space (U,E,τ)

(F,A)

and (G,B) there exist two soft open sets (H,C) and (K,D) such that $(F,A) \tilde{\subseteq}$

(H,C) $(G,B) \tilde{\subseteq} (K,D)$ and $(H,C) \prod (K,D) = \Phi$. A soft completely normal T_1 −space is called a soft T_5 −space.

Definition 2.12 ([27]) Let (X,E) and (Y,K) be soft classes and let $u : X \to Y$ and $p : E \to K$ be mappings. Then a mapping $f:\ (X,E) \to (Y,K)$ is defined as: for a soft set (F,A) in (X,E), $(f(F,A),B)$, $B = p(A) \subseteq K$ is a soft set in (Y,K) given by $f(F,A)(\beta) = u\left(\bigcup_{\alpha \in p^{-1}(\beta) \cap A} F(\alpha) \right)$ for $\beta \in K$. $(f(F,A),B)$ is called a soft image of a soft set (F,A). If B = K, then we shall write $(f(F,A),K)$ as $f(F,A)$.

Definition 2.13 ([27]) Let $f:\ (X,E) \to (Y,K)$ be a mapping from a soft class (X,E) to another soft class (Y,K) and (G,C) a soft set in soft class (Y,K) where $C \subseteq K$. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then $(f^{-1}(G,C),D)$, $D = p^{-1}(C)$, is a soft set in the soft classes (X,E) defined as: $f^{-1}(G,C)(\alpha) = u^{-1}(G(p(\alpha)))$ for $\alpha \in D \subseteq E$. $(f^{-1}(G,C),D)$, is called a soft inverse image of (G,C): Hereafter, we shall write $(f^{-1}(G,C),E)$ as $f^{-1}(G,C)$.

Theorem 2.14 ([27]) Let $f:(X,E) \to (Y,K)$, $u : X \to Y$ and $p : E \to K$ be mappings. Then for soft sets (F,A), (G,B) and a family of soft sets (F_i, A_i) in the soft class (X,E) we have:

(1) $f(\Phi) = \Phi$,

(2) $f((X,E)) = (Y,K)$,

(3) $f((F,A) \coprod (G,B)) = f((F,A)) \coprod f((G,B))$ in general $f(\coprod_{i \in I}(F_i, A_i)) = \coprod_{i \in I} f((F_i, A_i))$,

(4) $f((F,A) \prod (G,B)) \subseteq f((F,A)) \prod f((G,B))$ in general $f(\prod_{i \in I}(F_i, A_i)) \subseteq \prod_{i \in I} f((F_i, A_i))$,

(5) If $(F,A) \subseteq (G,B)$ then $f((F,A)) \subseteq f((G,B))$,

(6) $f^{-1}(\Phi) = \Phi$,

(7) $f^{-1}((Y,K))=(X,E)$,

(8) $f^{-1}((F,A)\coprod(G,B))=f^{-1}((F,A))\coprod f^{-1}((G,B))$ in general $f^{-1}(\coprod_{i\in I}(F_i,A_i))=\coprod_{i\in I}f^{-1}((F_i,A_i))$,

(9) $f^{-1}((F,A)\prod(G,B))=f^{-1}((F,A))\prod f^{-1}((G,B))$ in general $f^{-1}(\prod_{i\in I}(F_i,A_i))=\prod_{i\in I}f^{-1}((F_i,A_i))$,

(10) If $(F,A)\subseteq(G,B)$ then $f^{-1}((F,A))\subseteq f^{-1}((G,B))$.

Definition 2.15 ([6], [15]) Assume that U is an initial universe set and let E be a set of parameters. We denote to the family of all fuzzy subsets of U and $A\subseteq E$ by I^U. Then the mapping $F_A;A\to I^U$ defined by $F_A(e)=\mu_{F_A}^e$ (a fuzzy subset of U), is called a fuzzy soft set over (U,E), where $\mu_{F_A}^e=\overline{0}$ if $e\in E\setminus A$ and $\mu_{F_A}^e\neq\overline{0}$ if $e\in A$. The set of all fuzzy soft sets over (U,E) is written as $FS(U,E)$.

Definition 2.16 ([1]) **.** The fuzzy soft set $F_\phi\in FS(U,E)$ is said to be null fuzzy soft set and it is denoted by Φ, if for all $e\in E$, $F(e)$ is the null fuzzy set $\overline{0}$ of U, where $\overline{0}(x)=0$ for all $x\in U$.

Definition 2.17 ([1]) **.** Let $F_E\in FS(U,E)$ and $F_E(e)=\overline{1}$ for all $e\in E$, where $\overline{1}(x)=1$ for all $x\in U$. Then F_E is called absolute fuzzy soft set. It is denoted by $\overline{E}$.

Definition 2.18 ([23]) **.** A fuzzy soft set F_A is said to be a fuzzy soft subset of a fuzzy soft set G_B over a common universe U if $F_A(e)\subseteq G_B(e)$ for all $e\in E$, i.e., if $\mu_{F_A}^e(x)\le\mu_{G_B}^e(x)$ for all $x\in U$ and for all $e\in E$.

Definition 2.19 ([1]). For any pair of fuzzy soft sets F_A and G_B over a common universe U are called fuzzy soft equal if each one of them is a fuzzy soft subset of other.

F_e (U,E)

$F_e(a) = \mu_{F_e}$ $\mu_{F_e} \neq \overline{0}$ $a \neq e$

F_A (U,E) G_e

(U,E) $G_e \in F_A$

$\mu_{G_e} \subseteq \mu^e_{F_A} = F_A(e)$ $\mu_{G_e}(x) \leq \mu^e_{F_A}(x)$ $x \in U$

Definition 2.22 ([1]). The union of two fuzzy soft sets F_A and G_B over the common universe U is the fuzzy soft set H_C, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \bigcup B$. Here we write $H_C = F_A \vee G_B$.

Definition 2.23 ([1]) **.** Let F_A and G_B be two fuzzy soft set, then the intersection of F_A and G_B is a fuzzy soft set H_C, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \bigcap B$. Here we write $H_C = F_A \wedge G_B$.

Definition 2.24 ([17], [13]) **.** Assume that ψ is a collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψ such that the following axioms:

(i) Φ, $\overline{E}$ belong to ψ.

(ii) For any collection $\{F_{A_i}\}_{i \in I}$ of fuzzy soft sets in ψ, such that $\underset{i \in I}{\vee} F_{A_i}$ belongs to ψ.

(iii) For any pair of fuzzy soft sets in ψ, their intersection belongs to ψ.

Then (U,E,ψ) U. Further, any member in ψ U U. Moreover, a fuzzy soft set F_A is said to be a fuzzy soft semi open iff F_A subset of $cl(\mathrm{int}(F_A))$. Further, any fuzzy soft set is called fuzzy soft semi closed set in U iff its complement is fuzzy soft semi open.

(X,E,τ) (Y,T,σ)
(X,E,τ) (Y,T,σ)
$^{-1}$ G_B τ G_B σ

3. Soft Tychonoff Spaces and Some New Soft Separation Axioms

In this section, we introduce and study soft Tychonoff spaces and some new soft separation axioms in soft topological spaces.

Definition 3.1

Let (U,E,τ) be a soft topological space over U, then (U,E,τ) is a soft completely regular space if given any soft closed set (F,A) and any soft point $e_x \tilde{\notin} (F,A)$, then there is a soft continuous function $f:(U,E)\to(U,E)$ such that $f(e_x)=\Phi$ and, $f((F,A))=(U,E)$. In other terms, this condition says that e_x and (F,A) can be separated by a soft continuous function.

Definition 3.2

Let (U,E,τ) be a soft topological space over U, then (U,E,τ) is a soft Tychonoff space if (U,E,τ) is a soft completely regular and soft T_1-space.

Definition 3.3

A topological space X is called a soft public regular space iff for each pair consisting of soft point e_x and a soft closed set (F,A) disjoint from e_x, there exist two disjoint soft semi open sets (G,B) and (H,C) containing e_x and (F,A) respectively .

Example 3.4

Let $U=\{s_1,s_2\}$ e_1), modern(e_2
$(F,A),(G,B),(H,E)$ are three soft sets to

(U,E,τ) is a soft topological space, where $A=\{e_1\}$, $B=\{e_2\}$, $F(e_1)=\{s_1\}$, $G(e_2)=\{s_2\}$, $H(e_1)=\{s_1\}$, $H(e_2)=\{s_2\}$ and $\tau=$ $\{\Phi,(U,E),(F,A),(G,B),(H,E)\}$ $=$ $\{\Phi,$ $\{(e_1,U),(e_2,U)\},\{(e_1,s_1)\},\{(e_2,s_2)\},\{(e_1,s_1),(e_2,s_2)\}\}$. Then ($U$, τ, E) is soft topological space. We have $K_1=(U,E)$, $K_2=\Phi$, $K_3=\{(e_1,s_2),(e_2,U)\}$, $K_4=\{(e_1,U),(e_2,s_1)\}$, and $K_5=\{(e_1,s_2),(e_2,s_1)\}$ the all soft closed sets in (U,E), since $K_3 \tilde{\subseteq} cl(\text{int}(K_3))$ and $K_4 \tilde{\subseteq} cl(\text{int}(K_4))$, then K_3 and K_4 are soft semi open sets in (U,E), Also, K_1, K_2, (F,A) and (G,B) are soft semi open sets. It's clear there is no soft point $e_x=(e,x)$ satisfies $(e,x) \tilde{\notin} K_1$, also for any soft point we have $(e,x) \tilde{\notin} K_2$ and then we can take two disjoint soft semi open sets K_1 and K_2 where $(e,x) \tilde{\in} K_1$ and $K_2 \tilde{\subseteq} K_2$. Moreover, for each soft point $(e_1,s_1) \tilde{\notin} K_3$, $(e_2,s_2) \notin K_4$, $(e_1,s_1) \tilde{\notin} K_5$ and $(e_2,s_2) \notin K_5$ there exist two disjoint soft semi open sets such that:

$(e_1,s_1) \tilde{\in} (F,A),\ and\quad K_3 \tilde{\subseteq} K_3$
$(e_2,s_2) \tilde{\in} (G,B),\ and\quad K_4 \tilde{\subseteq} K_4$
$(e_1,s_1) \tilde{\in} (F,A),\ and\quad K_5 \tilde{\subseteq} K_3$
$(e_2,s_2) \tilde{\in} (G,B),\ and\quad K_5 \tilde{\subseteq} K_4$

Then (U,E,τ) is a soft public regular space.

Lemma 3.5:
Let (U,E,τ) be a soft regular topological space over U, then (U,E,τ) is a soft public regular space.

Proof:
Suppose (U,E,τ) (F,A) (U,E,τ)
chose e_x any soft point in (U,E) such that $e_x \tilde{\notin} (F,A)$
D and V in (U,E) $x \in D$ and $B \subseteq V$, but

D and V are semi open sets in (U,E) because each of (G,B) and (H,C) are soft open sets. Therefore (U,E,τ) is a soft public regular space.

Remark 3.6: The converse of lemma (3.5) is not true.

Example 3.7:
See example (3.4) where $U=\{s_1,s_2\}$, $E=\{e_1,e_2\}$, $A=\{e_1\}$, $B=\{e_2\}$, $F(e_1)=\{s_1\}$, $G(e_2)=\{s_2\}$, $H(e_1)=\{s_1\}$, $H(e_2)=\{s_2\}$ and $\tau=\{\Phi,(X,E),(F,A),(G,B),(H,E)\}$ we have (U,E) is soft public regular space but is not soft regular since $(e_2,s_2)\notin K_4$, and (U,E) does not have two disjoint soft open sets D and V in (U,E) such that $(e_2,s_2)\tilde{\in} D$ and $K_4 \subseteq V$.

Definition 3.8:
Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is a soft $PR-T_3$ iff (U,E,τ) is soft public regular and soft T_1 −space.

Example 3.9:
Let $U=\{a_1,\ a_2,\ a_3\}$ be the set of students under consideration. Let $E=\{$ pleasing personality (e_1); conduct (e_2); good result (e_3); sincerity (e_4)$\}$ be the set of parameters framed to choose the best student. Suppose that the soft set (F,A) describing the Mr. X opinion to choose the best student of an academic year was defined by

$A=\{e_1,e_2\}$

$F(e_1)=\{a_1\}$, $F(e_2)=\{a_1,\ a_2,\ a_3\}$

In addition, we assume that the “best student” in the opinion of another teacher, say Mr.Y, is described by the soft set (G,B), where

$B=\{e_1,e_3,e_4\}$

$G(e_1)=\{a_2,a_3\}$, $G(e_3)=\{a_1,\ a_2,\ a_3\}$, $G(e_4)=\{a_1,\ a_2,\ a_3\}$

Consider that:
. Then (U,E,τ) is a soft public regular and soft $\tau=\{\Phi,(U,E),(F,A),(G,B)\}$ $-T_1$, thus (U,E,τ) is a soft $PR-T_3$ space.

Lemma 3.10: Every soft T_3-space is soft $PR-T_3$ space.

Proof: Let (U,E,τ) be a soft T_3-space we have (U,E,τ) is soft T_1-space and soft regular, so (U,E,τ) is a soft public regular by lemma (3.5), therefore (U,E,τ) is soft $PR-T_3$.

Remark 3.11: From (3.6) we have, if (U,E,τ) is soft $PR-T_3$. Then it is may not be soft T_3-space.

Remark 3.12: It is clear that every soft T_1-space is a soft semi T_1-space but the converse is not true.

Definition 3.13:
Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is a soft $SPR-T_3$ iff (U,E,τ) is soft public regular and soft semi T_1-space.

Example 3.14: Let $U=\{s_1,s_2\}$, $E=\{e_1,e_2\}$, $A=\{e_1\}$, $B=\{e_2\}$, $F(e_1)=\{s_1\}$, $G(e_2)=\{s_2\}$, $H(e_1)=\{s_1\}$, $H(e_2)=\{s_2\}$ and $\tau=\{\Phi,(U,E),(F,A),(G,B),(H,E)\}$. we consider that (U,E,τ) is soft public regular and soft semi T_1, therefore (U,E,τ) is soft $SPR-T_3$.

Lemma 3.15: A soft topological space (U,E,τ) is a soft $SPR-T_3$ if (U,E,τ) is soft $PR-T_3$

Proof: Let (U,E,τ) be a soft $PR-T_3$, then (U,E,τ) is soft public regular and soft T_1 but every soft T_1 is soft semi T_1, then (U,E,τ) is soft $SPR-T_3$.

Remark 3.16: The converse of lemma (3.15) is not true.

Example 3.17: See example (3.14) we have (U,E,τ) T_1
$(e_1,s_1)\neq(e_1,s_2)$ in (U,E) and (U,E) has

not two disjoint soft open sets D and V such that $(e_1,s_2)\,\tilde{\in}\,V$, $(e_1,s_1)\,\tilde{\notin}\,V$ and $(e_1,s_1)\,\tilde{\in}\,D$, $(e_1,s_2)\,\tilde{\notin}\,D$.

Definition 3.18:
A soft topological space (U,E,τ) is called a soft strongly completely regular space iff for each pair consisting of soft point e_x and soft semi closed set (F,A) disjoint from e_x, there exists a soft continuous function $f:(U,E)\to(U,E)$ such that $f((F,A))=(U,E)$ and $f(e_x)=\Phi$.

Lemma 3.19: Every soft strongly completely regular is soft completely regular.

Proof: Suppose that (U,E,τ) is a soft strongly completely regular space. Now for each $e_x\,\tilde{\notin}\,(G,B)$ where (G,B) is a soft closed set, we have (G,B) is a soft semi closed set, so there exists a soft continuous function $f:(U,E)\to(U,E)$ such that $f((F,A))=(U,E)$ and $f(e_x)=\Phi$, then (U,E,τ) is a soft completely regular space.

Definition 3.20:
Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is a soft $SCR-T_3$ iff (U,E,τ) is a soft strongly completely regular and soft T_1-space.

Lemma 3.21:
A soft topological space (U,E,τ) is a soft Tychonoff if (U,E,τ) is soft $SCR-T_3$.

Proof: Suppose that (U,E,τ) is $SCR-T_3$ then (U,E,τ) is a soft strongly completely regular and soft T_1 −space. Thus (U,E,τ) is a soft completely regular by lemma (3.19). Then (U,E,τ) is a soft Tychonoff space.

Definition 3.22:

Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is a soft S^2CR-T_3 iff (U,E,τ) is soft strongly completely regular and soft semi T_1-space.

Lemma 3.23: A soft topological space (U,E,τ) is soft S^2CR-T_3 if (U,E,τ) is soft $SCR-T_3$.

Proof: Let (U,E,τ) be a soft $SCR-T_3$ then (U,E,τ) is a soft strongly completely regular and soft T_1 but every soft T_1- space is a soft semi T_1-space then (U,E,τ) is soft S^2CR-T_3.

Definition 3.24: Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is a soft public normal space iff for each two disjoint soft closed sets (G,B) and (H,C) in (U,E,τ), there exist two disjoint soft semi open sets in (U,E) containing (G,B) and (H,C), respectively.

Example 3.25:

Let $U=\{s_1,s_2,s_3,s_4\}$, $E=\{e_1,e_2,e_3,e_4\}$, $A=\{e_1\}$, $B=\{e_2\}$, $C=\{e_1,e_3,e_4\}$, $D=\{e_1,e_2,e_3\}$, $M=\{e_1,e_2\}$, $F(e_1)=\{s_1\}$, $G(e_2)=\{s_2\}$, $H(e_1)=\{s_1,s_3\}$, $H(e_3)=\{s_2,s_4\}$, $H(e_4)=U$, $K(e_1)=\{s_1,s_2,s_4\}$, $K(e_2)=U$, $K(e_3)=\{s_1,s_3\}$, $T(e_1)=\{s_1,s_3\}$, $T(e_2)=\{s_2\}$, $T(e_3)=\{s_2,s_4\}$, $T(e_4)=U$, $L(e_1)=\{s_1\}$, $L(e_2)=\{s_2\}$ and $\tau=\{\Phi,(U,E),(F,A),(G,B),(H,C),(K,D),(T,E),(L,M)\}$. Then (U,E,τ) is soft topological space . We consider that

$W_1=(U,E)$,

$W_2=\{(e_1,\{s_2,s_3,s_4\}),(e_2,U),(e_3,U),(e_4,U)\}$, $W_3=\{(e_1,U),(e_2,\{s_1,s_3,s_4\}),(e_3,U),(e_4,U)\}$,

$W_4=\{(e_1,\{s_2,s_4\}),(e_2,U),(e_3,\{s_1,s_3\})\}$, $W_5=\{(e_1,s_3),(e_2,\{s_2,s_4\}),(e_4,U)\}$,

$W_6=\{(e_1,\{s_2,s_3,s_4\}),(e_2,\{s_1,s_3,s_4\}),(e_3,U),(e_4,U)\}$, $W_7=\{(e_1,\{s_2,s_4\}),(e_2,\{s_1,s_3,s_4\}),(e_3$

and $W_8=\Phi$ the all soft closed sets in (U,E). Then the following are considered:

1)- $W_8\sqcap W_i=\Phi$; $(i=1,2,3,...,7)$

2)- $W_4 \prod W_5 = \Phi$

3)- $W_5 \prod W_7 = \Phi$

Now, since (U,E), Φ and (H,C) are soft open sets in (U,E,τ), then (U,E), Φ and (H,C) are soft semi open sets in (U,E,τ), also since $(G,B) \widetilde{\subseteq} W_4 \widetilde{\subseteq} cl((G,B)) \Rightarrow W_4 \widetilde{\subseteq} cl(\text{int}(W_4)))$, then W_4 is soft semi open set in (U,E,τ).

If (1) satisfy, we can take (U,E) and Φ two disjoint soft semi open sets such that: $W_8 \widetilde{\subseteq} \Phi$ and $W_i \widetilde{\subseteq} (U,E)$;$(i = 1,2,3,...,7)$

If (2) satisfies, we can take W_4 and (H,C) two disjoint soft semi open sets such that: $W_4 \widetilde{\subseteq} W_4$ and $W_5 \widetilde{\subseteq} (H,C)$

If (3) satisfies, we can take W_4 and (H,C) two disjoint soft semi open sets such that: $W_7 \widetilde{\subseteq} W_4$ and $W_5 \widetilde{\subseteq} (H,C)$, then (U,E,τ) is a soft public normal space.

Remark 3.26:
It is clear that every soft normal space is a soft public normal space but the converse is not true.

Example 3.27:
See example (3.25) where $U = \{s_1, s_2, s_3, s_4\}$, $E = \{e_1, e_2, e_3, e_4\}$, and $\tau = \{\Phi, (U,E), (F,A), \quad (G,B), (H,C), (K,D), (T,E), (L,M)\}$ we have $W_4 \prod W_5 = \Phi$. But (U,E) does not have V_1, V_2 two disjoint soft open sets such that $W_4 \widetilde{\subseteq} V_1$ and $W_5 \widetilde{\subseteq} V_2$.

Definition 3.28: Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is a soft $PN - T_4$ iff (U,E,τ) is soft public normal and soft T_1.

Lemma 3.29: Every soft T_4- space is a soft $PN - T_4$ space.

Proof: Let (U,E,τ) be a soft T_4 -space we have (U,E,τ) is soft T_1-space and soft normal. Hence (U,E,τ) is a soft public normal by (3.26), therefore (U,E,τ) is soft $PN-T_4$.

Remark 3.30: From (3.26) we have, if (U,E,τ) is a soft $PN-T_4$. Then it is may not be soft T_4.

Definition 3.31: Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is soft $SPN-T_4$ iff (U,E,τ) is soft public normal and soft semi T_1.

Lemma 3.32: A soft topological space (U,E,τ) is soft $SPN-T_4$ if (U,E,τ) is soft $PN-T_4$.

Definition 3.33: A soft topological space (U,E,τ) is a soft semi completely normal space iff for all two non-empty separated soft sets (F,A) and (G,B) there exist two soft semi open sets (H,C) and (K,D) such that $(F,A)\tilde{\subseteq}$ (H,C), $(G,B)\tilde{\subseteq}(K,D)$ and $(H,C)\prod(K,D)=\Phi$.

Remark 3.34: It is clear that every soft completely normal space is soft semi completely normal space.

Definition 3.35: Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is soft $SCN-T_5$ iff (U,E,τ) is soft semi completely normal and soft T_1.

Lemma 3.36: Every soft T_5 -space is soft $SCN-T_5$ space.

Proof: Let (U,E,τ) be a soft T_5 -space we have (U,E,τ) is soft T_1-space and soft completely normal but by (3.34) we have (U,E,τ) is a soft semi completely normal space, then (U,E,τ) is soft $SCN-T_5$.

Definition 3.37: Let (U,E,τ) be a soft topological space, then we say that (U,E,τ) is soft S^2CN-T_5 iff (U,E,τ) is soft semi completely normal and soft semi T_1.

1.38 Lemma: Every soft $SCN-T_5$ space is soft S^2CN-T_5 space.

Proof: Suppose (U,E,τ) is a soft $SCN-T_5$, then (U,E,τ) is soft semi completely normal and soft T_1 but every soft T_1 is soft semi T_1, therefore we have (U,E,τ) is soft S^2CN-T_5.

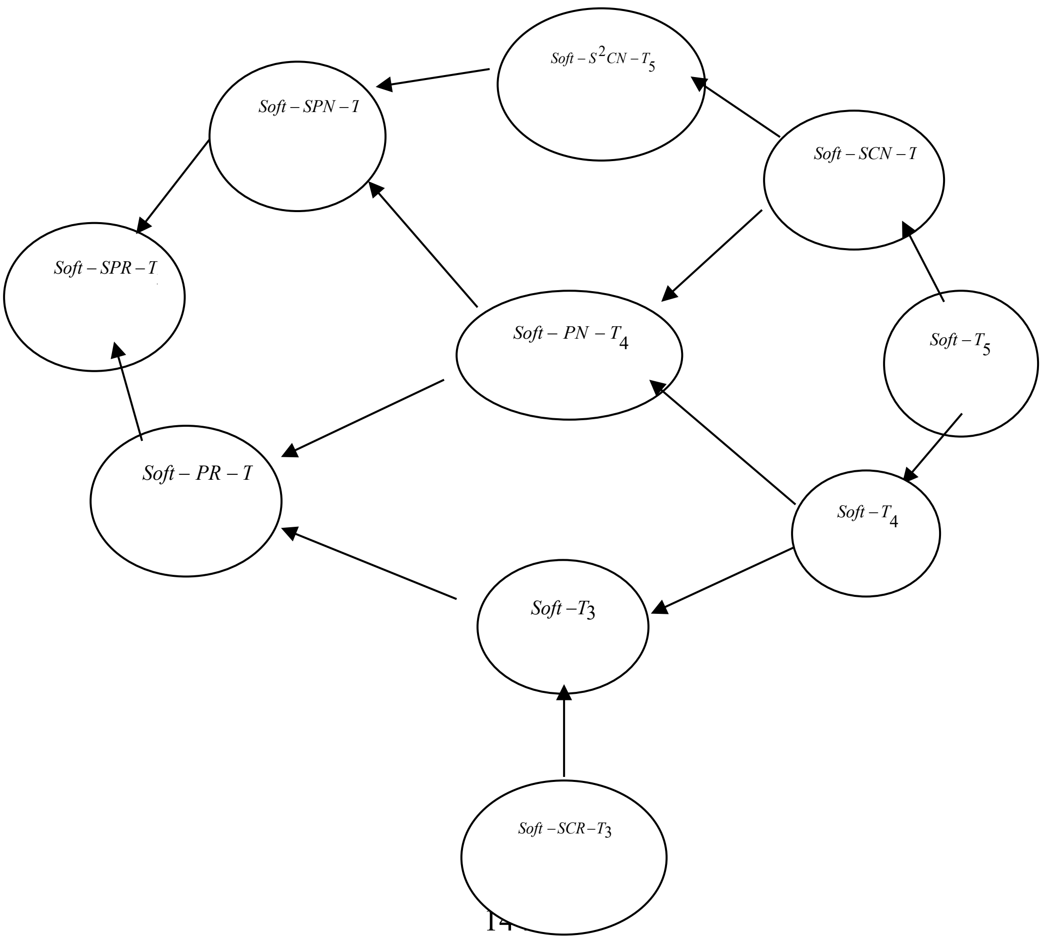
Lemma 3.39: Every soft semi completely normal space is soft public normal.

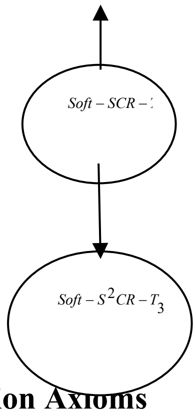
Proof: Suppose that (U,E,τ) is a soft semi completely normal space and (H,C), (K,D) are two disjoint soft closed sets, thus $(H,C)\prod(K,D)=\Phi$, but $(H,C)=cl((H,C))$ and $(K,D)=cl((K,D))\Rightarrow(H,C)\prod cl((K,D))=\Phi$ and $cl((H,C))\prod(K,D)=\Phi$. Then $(U,E)=(H,C)\,/\,(K,D)\Rightarrow$ there exist two disjoint soft semi open sets (H',C'), (K',D') such that $(H,C)\tilde{\subseteq}(H',C')$ and $(K,D)\tilde{\subseteq}(K',D')$, then (U,E,τ) is soft public normal.

Lemma 3.40: Every soft $PN-T_4$ space is soft $PR-T_3$ space.

Proof: Suppose that (U,E,τ) is a soft $PN-T_4$ space and e_x is a soft point in (U,E) such that $e_x\tilde{\notin}(G,B)$, where (G,B) is soft closed set in (U,E,τ), we have e_x soft closed set in (U,E,τ) (since (U,E,τ) is soft T_1) also e_x $\prod(G,B)=\Phi$ but (U,E,τ) is soft public normal space, then there exist two disjoint soft semi open sets (H,C), (K,D) such that $e_x\tilde{\in}(H,C)$ and $(G,B)\tilde{\subseteq}(K,D)$. Then (U,E,τ) is soft $PR-T_3$.

Remark 3.41: see the following diagram:





4. Some New Fuzzy Soft Separation Axioms

In this section, we will introduce some new concepts on fuzzy soft separation axioms to open the door for other researchers to study these new concepts and find more of fuzzy soft separation axioms in future work.

4.1 Definition:

Let (U,E,ψ) be a fuzzy soft topological space. Then (U,E,ψ) is called a fuzzy soft public regular iff for each pair consisting of fuzzy soft point F_e and a fuzzy soft closed set G_B satisfies $\mu^e_{G_B}(x) < \mu^e_{F_e}(x) = \alpha \in (0,1]$, $\forall\, x \in U \,\&\, e \in E$ there exist disjoint fuzzy soft semi open sets K_D and H_C such that $\alpha < \mu^e_{K_D}(x)$ and $\mu^e_{G_B}(x) \le \mu^e_{H_C}(x)$ for all $x \in U \,\&\, e \in E$ ($FSPR$, for short).

4.2 Definition:

Let (U,E,ψ) be a fuzzy soft topological space. Then (U,E,ψ) is called a fuzzy soft strongly completely regular iff for each pair consisting of fuzzy soft point F_e and a fuzzy soft semi closed set G_B satisfies $\mu^e_{G_B}(x) < \mu^e_{F_e}(x) = \alpha \in (0,1], \forall\, x \in U \,\&\, e \in E$, there exists a fuzzy soft continuous function $f:(U,E) \to (U,E)$ such that $f(G_B) = \overline{E}$ and $f(F_e) = \overline{0}$. ($FSSCR$, for short).

4.3 Definition:

Let (U,E,ψ) be a fuzzy soft topological space. Then (U,E,ψ) is called a fuzzy soft public normal space iff for each disjoint fuzzy soft closed sets F_A and G_B in (U,E,ψ), there exist two disjoint fuzzy soft semi open sets H_C and K_D such that $\mu^e_{F_A}(x) \le \mu^e_{K_D}(x)$ and $\mu^e_{G_B}(x) \le \mu^e_{H_C}(x)$ for all $x \in U$ & $e \in E$. ($FSPN$, for short).

4.4 Definition:
Let (U,E,ψ) be a fuzzy soft topological space. Then (U,E,ψ) is called a fuzzy soft semi completely normal space ($FSSCN$, for short) iff for each $\overline{E} = F_A / G_B$, there exist two disjoint fuzzy soft semi open sets H_C and K_D such that $\mu^e_{F_A}(x) \le \mu^e_{K_D}(x)$ and $\mu^e_{G_B}(x) \le \mu^e_{H_C}(x)$ for all $x \in U$ & $e \in E$.

5. Concluding Remarks

Let (U,E,ψ) be a fuzzy soft topological space. Then by present research in the current work, the following are interesting questions for future paper consideration:

1. What is the possible relationships which considered between each pair of the following fuzzy soft spaces ($FSPR$, $FSSCR$, $FSPN$, and $FSSCN$).

2. What is the possible new types of fuzzy soft spaces can be obtained that coincided with the following soft spaces :
   $PR-T_3, SCR-T_3, PN-T_4, SCN-T_5, S^2CR-T_3, SPR-T_3$, $SPN-T_4$, and S^2CN-T_5.

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